

Structure Functions at Low Q^2 : Target Mass Corrections

Wally Melnitchouk

Jefferson Lab



■ Intriguing phenomena have been observed in low Q^2 structure functions

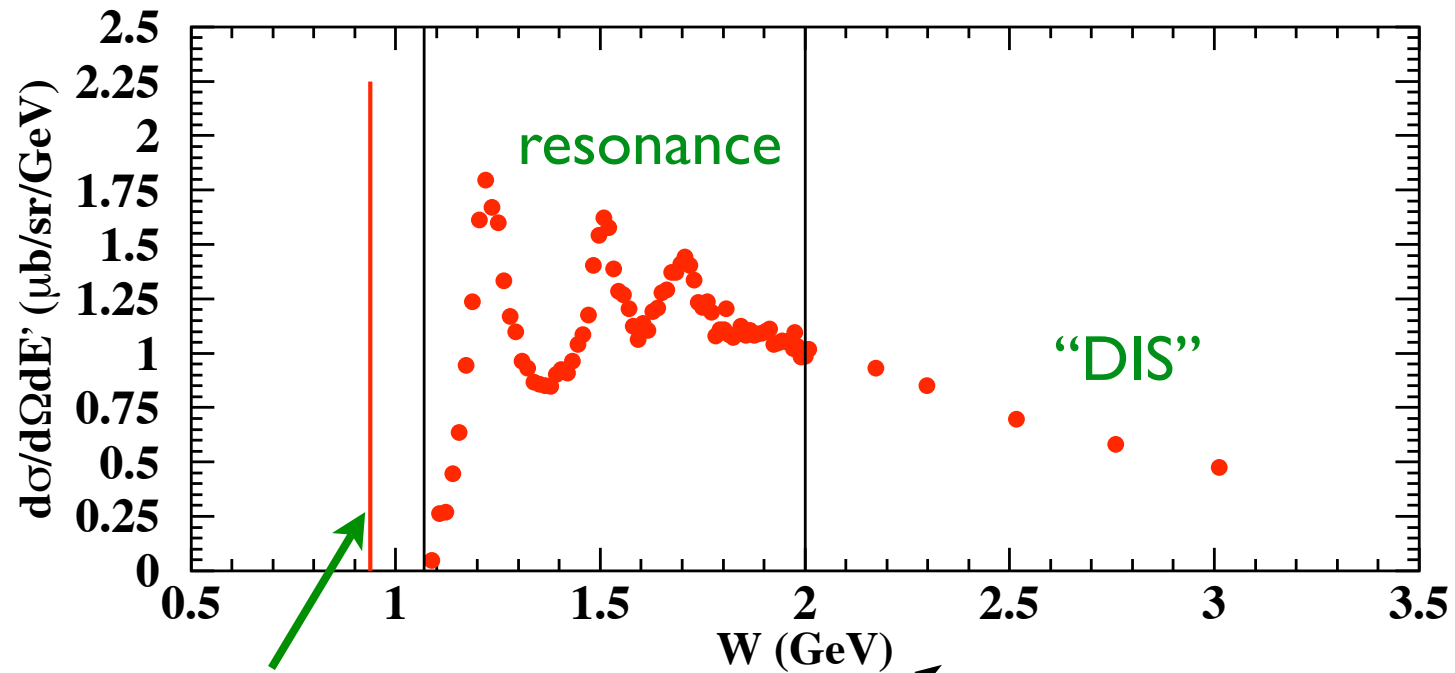
- ➡ quark-hadron (Bloom-Gilman) duality
- ➡ surprisingly small higher twist effects in (low) moments of structure functions
- ➡ anomalous approach of structure functions (*e.g.* R ratio) to real photon limit

■ Identification of correct low Q^2 dynamics requires careful treatment of “trivial” kinematical corrections

- ➡ target mass corrections
- ➡ related to behavior of structure functions in large- x limit
- ➡ focus mainly on e scattering; results carry over to ν case

Resonance-DIS transition

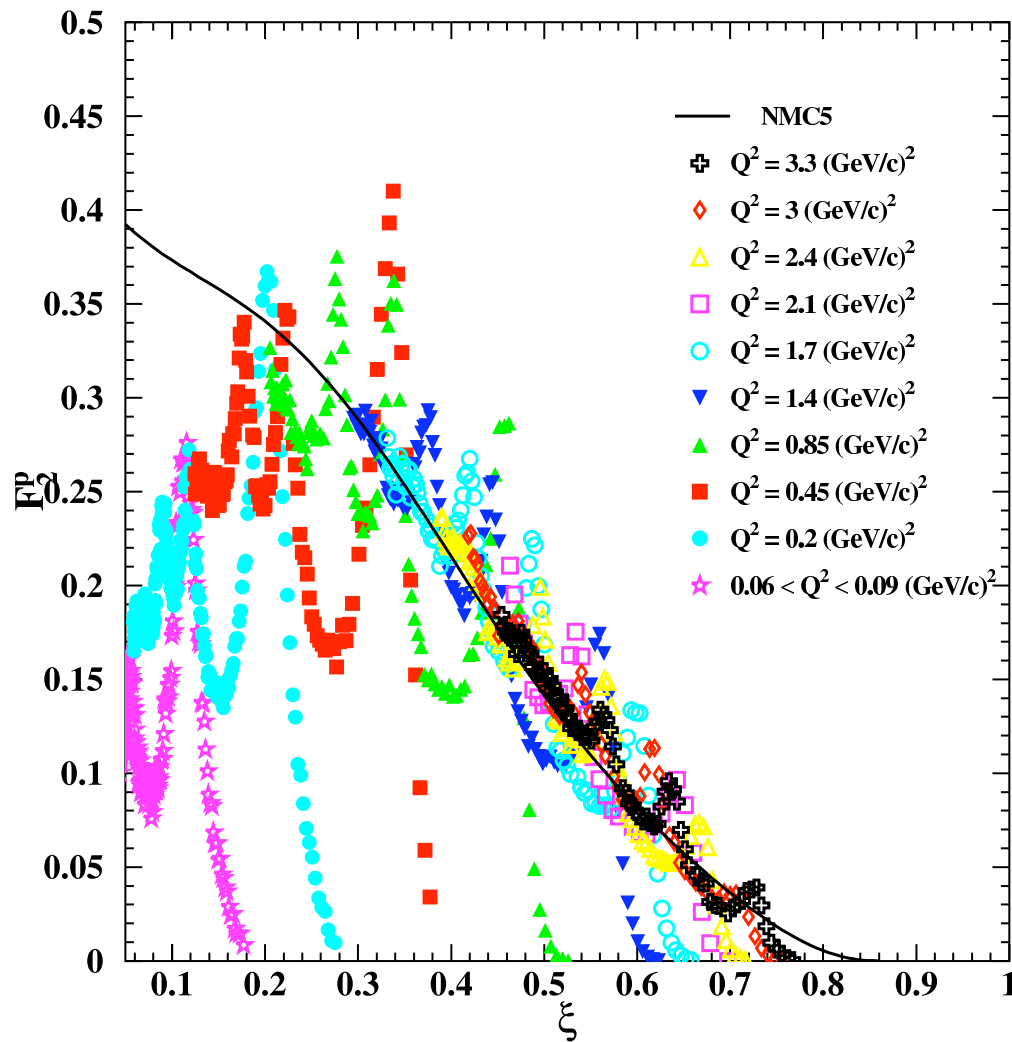
- As W decreases, DIS region gives way to region dominated by nucleon resonances



$$x = \frac{Q^2}{W^2 - M^2 + Q^2}$$

$W^2 = (p_N + q)^2$
 $= M^2 + 2M\nu - Q^2$
hadronic final state mass

Resonance-DIS transition



Average over
(strongly Q^2 dependent)
resonances
 \approx Q^2 independent
scaling function

“Bloom-Gilman duality”

Jefferson Lab (Hall C)

Niculescu et al., Phys. Rev. Lett. 85 (2000) 1182

Duality in QCD

Operator product expansion

$$\begin{aligned}
 & \int d^4x \, e^{iq \cdot x} \langle N | T(J^\mu(x) J^\nu(0)) | N \rangle \\
 &= \sum_k \left(-g^{\mu\nu} q^{\mu_1} q^{\mu_2} + g^{\mu\mu_1} q^\nu q^{\mu_2} + q^\mu q^{\mu_1} g^{\nu\mu_2} + g^{\mu\mu_1} g^{\nu\mu_2} Q^2 \right) \\
 & \quad \times q^{\mu_3} \cdots q^{\mu_{2k}} \underbrace{\frac{2^{2k}}{Q^{4k}} A_{2k} \Pi_{\mu_1 \cdots \mu_{2k}}}_{\langle N | \mathcal{O}_{\mu_1 \cdots \mu_{2k}} | N \rangle}
 \end{aligned}$$

local operators

$$\Pi_{\mu_1 \cdots \mu_{2k}} = p_{\mu_1} \cdots p_{\mu_{2k}} - (g_{\mu_i \mu_j} \text{ terms})$$

Georgi, Politzer (1976)

$$= \sum_{j=0}^k (-1)^j \frac{(2k-j)!}{2^j (2k)^j} g \cdots g \, p \cdots p$$

traceless, symmetric
rank-2k tensor

Duality in QCD

Operator product expansion

→ expand moments of structure functions
in powers of $1/Q^2$

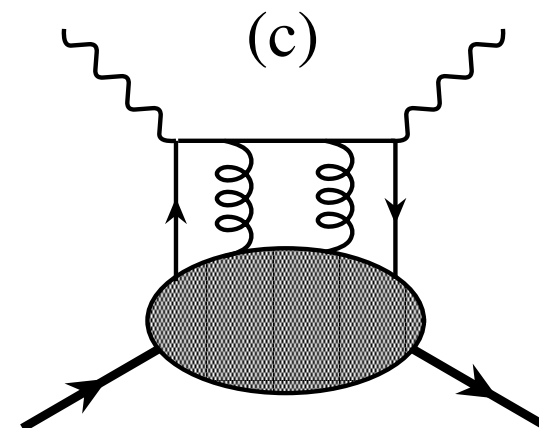
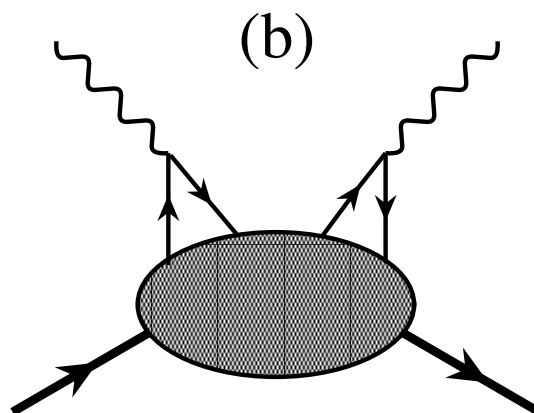
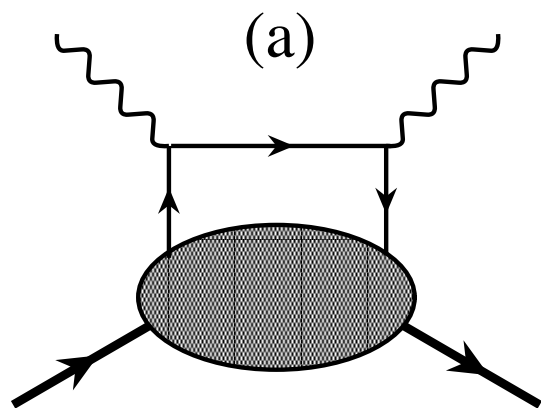
$$\begin{aligned} M_n(Q^2) &= \int_0^1 dx \, x^{n-2} F_2(x, Q^2) \\ &= A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \dots \end{aligned}$$

matrix elements of operators with
specific “twist” τ

$\tau = \text{dimension} - \text{spin}$

leading twist

higher twist



$$\tau = 2$$

$$\tau > 2$$

single quark
scattering

qq and *qg*
correlations

Duality in QCD

Operator product expansion

→ expand moments of structure functions
in powers of $1/Q^2$

$$\begin{aligned} M_n(Q^2) &= \int_0^1 dx \, x^{n-2} F_2(x, Q^2) \\ &= A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \dots \end{aligned}$$

If moment \approx independent of Q^2

→ higher twist terms $A_n^{(\tau>2)}$ small

Duality in QCD

Operator product expansion

→ expand moments of structure functions
in powers of $1/Q^2$

$$\begin{aligned} M_n(Q^2) &= \int_0^1 dx \, x^{n-2} F_2(x, Q^2) \\ &= A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \dots \end{aligned}$$

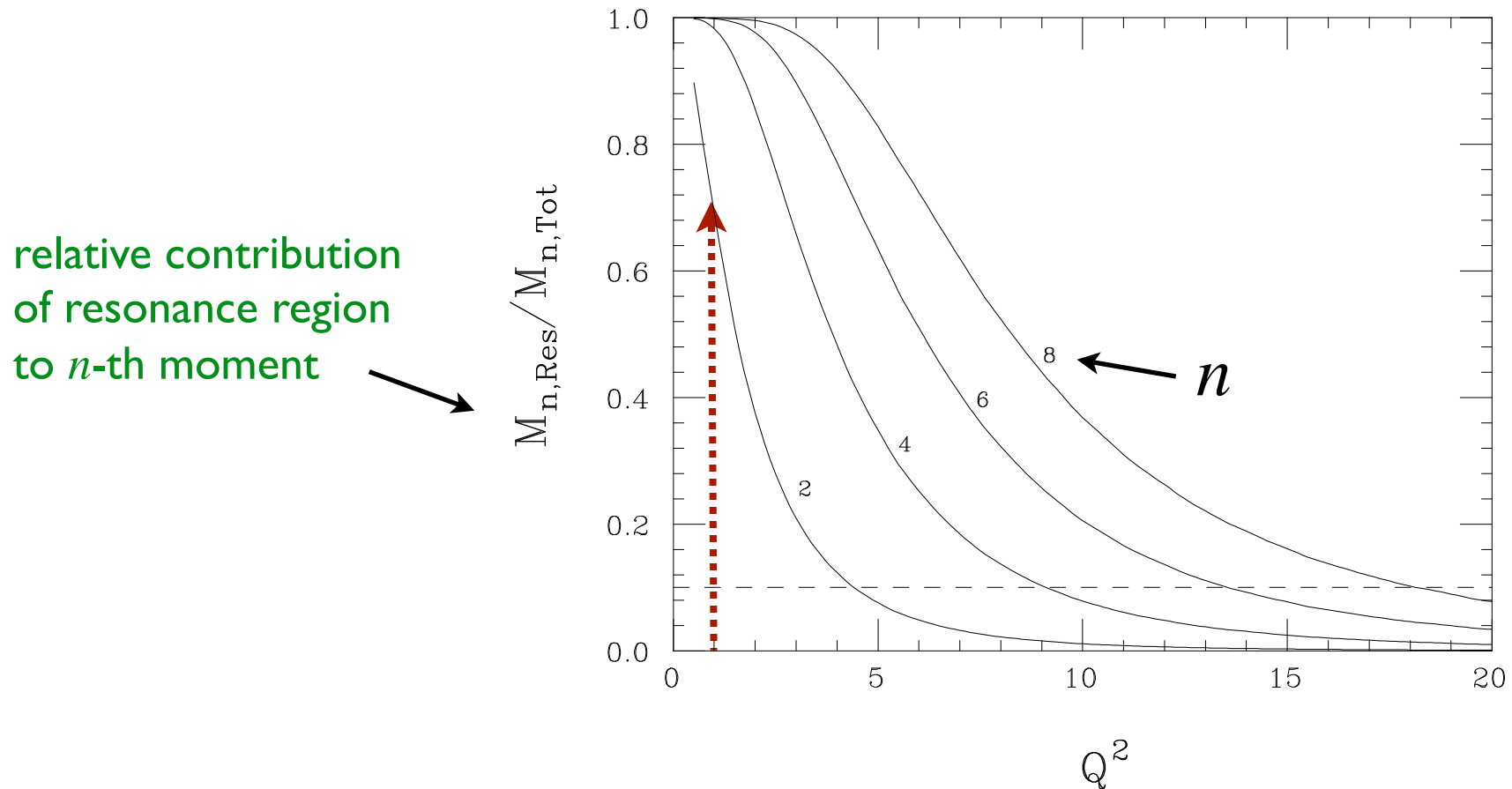
Duality \Longleftrightarrow suppression of higher twists

*de Rujula, Georgi, Politzer,
Ann. Phys. 103 (1975) 315*

Duality in QCD

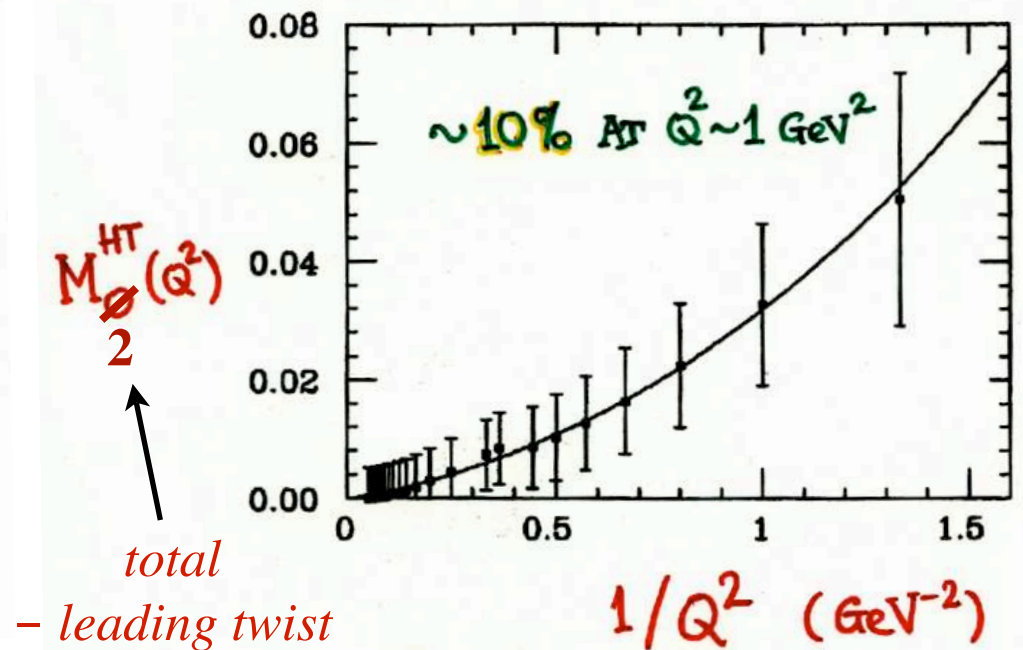
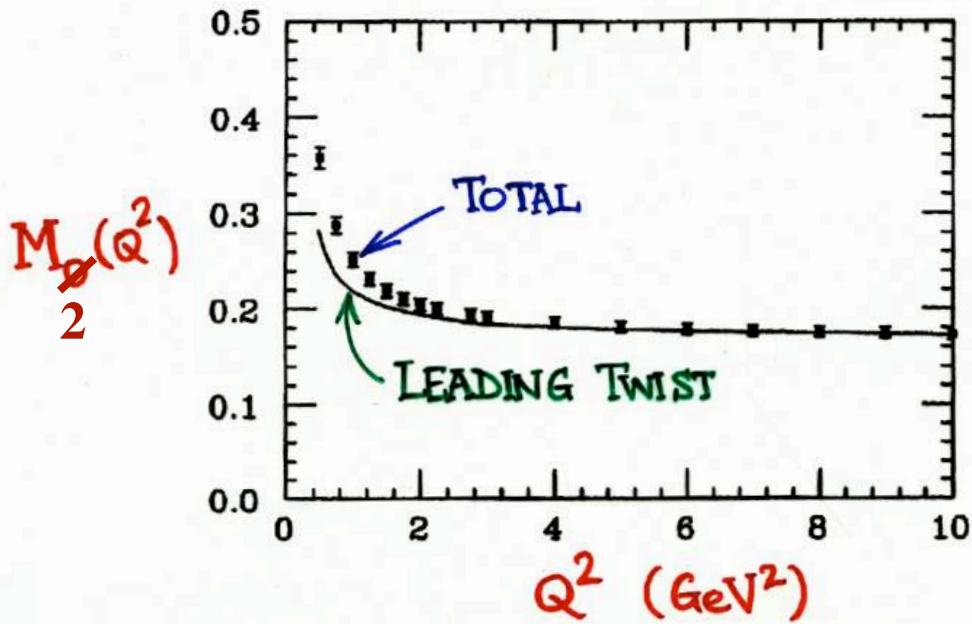
- Considerable data exists in resonance region, $W < 2 \text{ GeV}$
 - *common wisdom*: pQCD analysis not valid in resonance region
 - *in fact*: partonic interpretation of moments does include resonance region
- Resonances are an integral part of deep inelastic structure functions!
 - implicit role of quark-hadron duality

Proton F_2 moments



➡ At $Q^2 = 1 \text{ GeV}^2$, $\sim \underline{70\%}$ of lowest moment of F_2^p comes from $W < 2 \text{ GeV}$

Proton F_2 moments

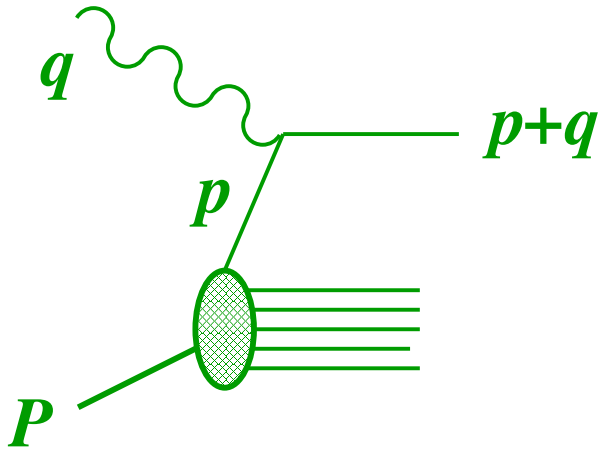


➔ BUT resonances and DIS continuum conspire to produce only $\sim 10\%$ higher twist contribution!

Nachtmann moments

- Formulation in terms of usual (“Cornwall-Norton”) moments mixes operators of same twist, but different spin, n
 - irrelevant at large Q^2 , but important at intermediate $Q^2/\nu^2 = 4M^2x^2/Q^2$
 - “target mass corrections” associated with higher spin operators (trace terms in OPE)
- Nachtmann (1973) constructed moments in which only operators with spin n contribute to the $n-2$ moment of structure function
 - automatically accounts for kinematical finite M^2/Q^2 effects

Parton kinematics



$$(p + q)^2 = m_q^2 \quad \left\{ \begin{array}{l} m_q = 0 \\ p_T = 0 \end{array} \right.$$

light-cone fraction of target's momentum carried by parton

$$\xi = \frac{p^+}{P^+} = \frac{p^0 + p^z}{M} \quad \text{Nachtmann scaling variable}$$

$$\Rightarrow \quad \xi = \frac{2x}{1+r}, \quad r = \sqrt{1 + 4M^2 x^2 / Q^2}$$

$$\rightarrow x \quad \text{as} \quad Q^2 \rightarrow \infty$$

■ n -th Nachtmann moment of F_2 structure function

$$\mu_2^n(Q^2) = \int_0^1 dx \frac{\xi^{n+1}}{x^3} \left(\frac{3 + 3(n+1)r + n(n+2)r^2}{(n+2)(n+3)} \right) F_2(x, Q^2)$$

→ n -th moment of PDFs at finite Q^2

■ Relate Nachtmann and CN moments

$$\begin{aligned} \mu_2^n(Q^2) = & M_2^n(Q^2) - \frac{n(n-1)}{n+2} \frac{M^2}{Q^2} M_2^{n+2}(Q^2) \\ & + \frac{n(n^2-1)}{2(n+3)} \frac{M^4}{Q^4} M_2^{n+4}(Q^2) - \frac{n(n^2-1)}{6} \frac{M^6}{Q^6} M_2^{n+6}(Q^2) + \dots \end{aligned}$$

→ mixing between lower & higher CN moments

- n -th Cornwall-Norton moment of F_2 structure function

$$M_2^n(Q^2) = \int dx \, x^{n-2} F_2(x, Q^2)$$
$$= \sum_{j=0}^{\infty} \left(\frac{M^2}{Q^2} \right)^j \frac{(n+j)!}{j!(n-2)!} \frac{A_{n+2j}}{(n+2j)(n+2j-1)}$$

→ $A_n = \int_0^1 dy \, y^n F(y)$ “quark distribution function”

$$F(y) \equiv \frac{F_2(y)}{y^2}$$

■ take inverse Mellin transform (+ tedious manipulations)

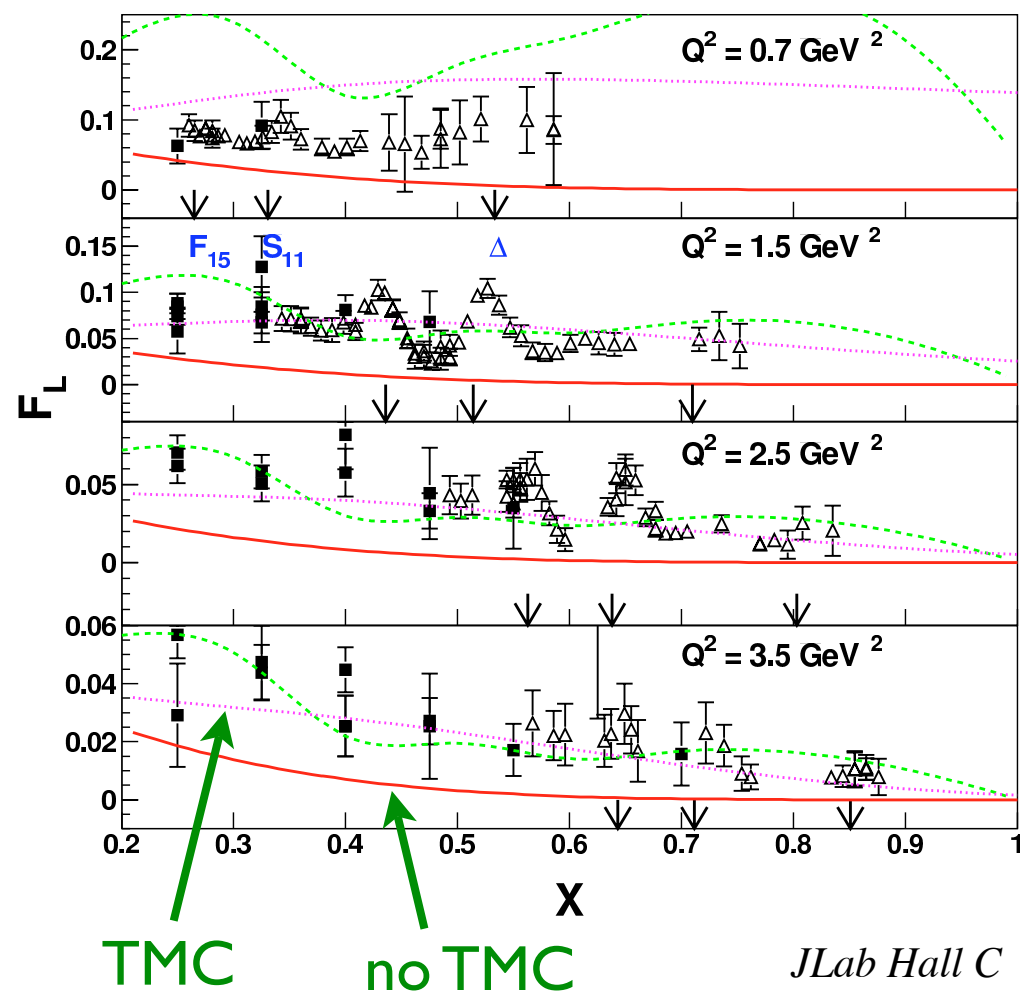
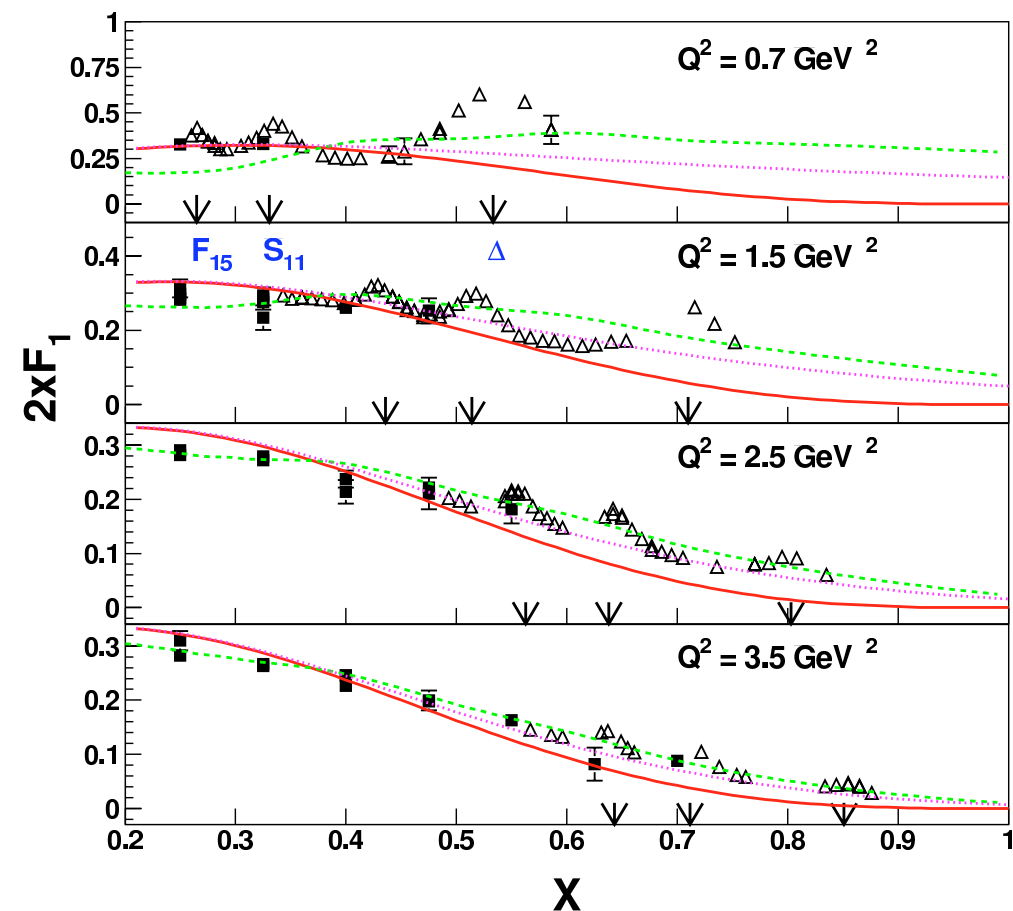
➡ target mass corrected structure function

$$F_2^{\text{GP}}(x, Q^2) = \frac{x^2}{r^3} F(\xi) + 6 \frac{M^2}{Q^2} \frac{x^3}{r^4} \int_{\xi}^1 d\xi' F(\xi') \\ + 12 \frac{M^4}{Q^4} \frac{x^4}{r^5} \int_{\xi}^1 d\xi' \int_{\xi'}^1 d\xi'' F(\xi'')$$

$$\xi = \frac{2x}{1+r} \qquad r = \sqrt{1 + 4x^2 M^2 / Q^2}$$

... similarly for other structure functions F_1, F_L

■ numerically...

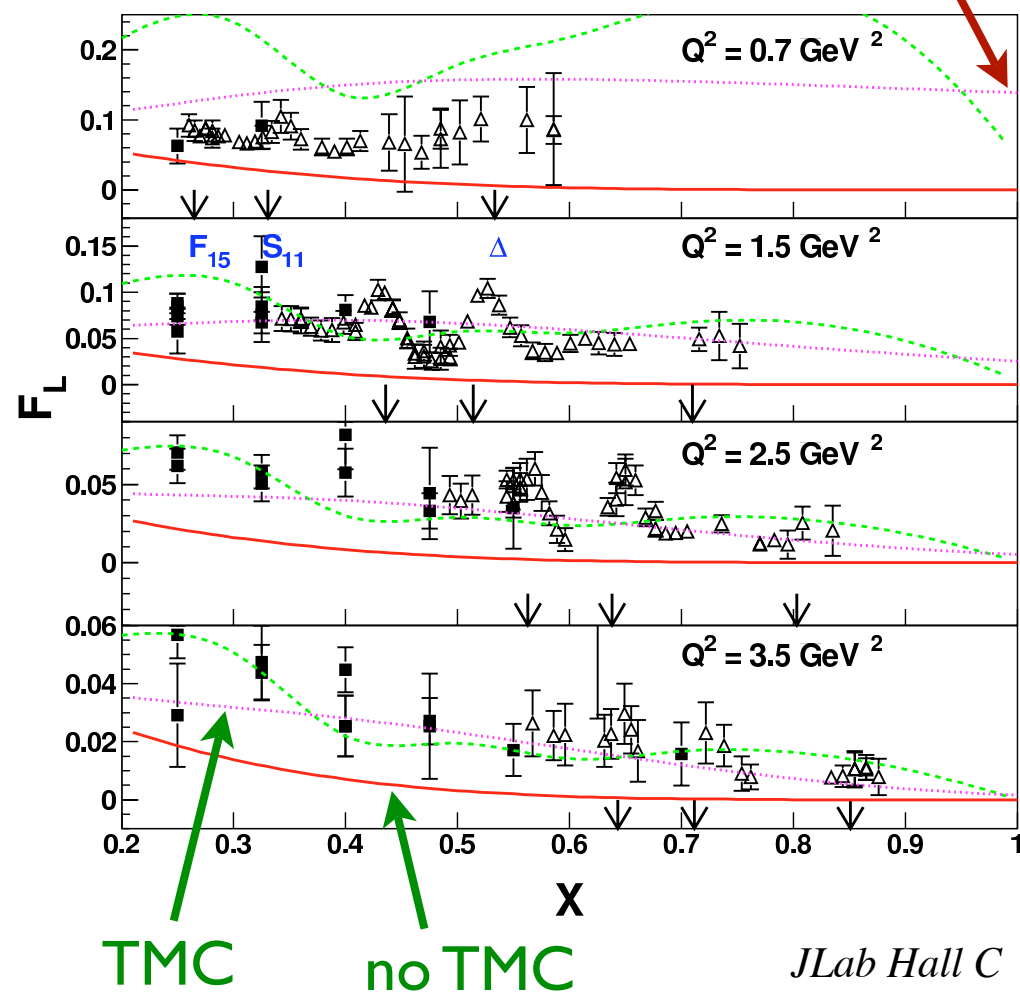
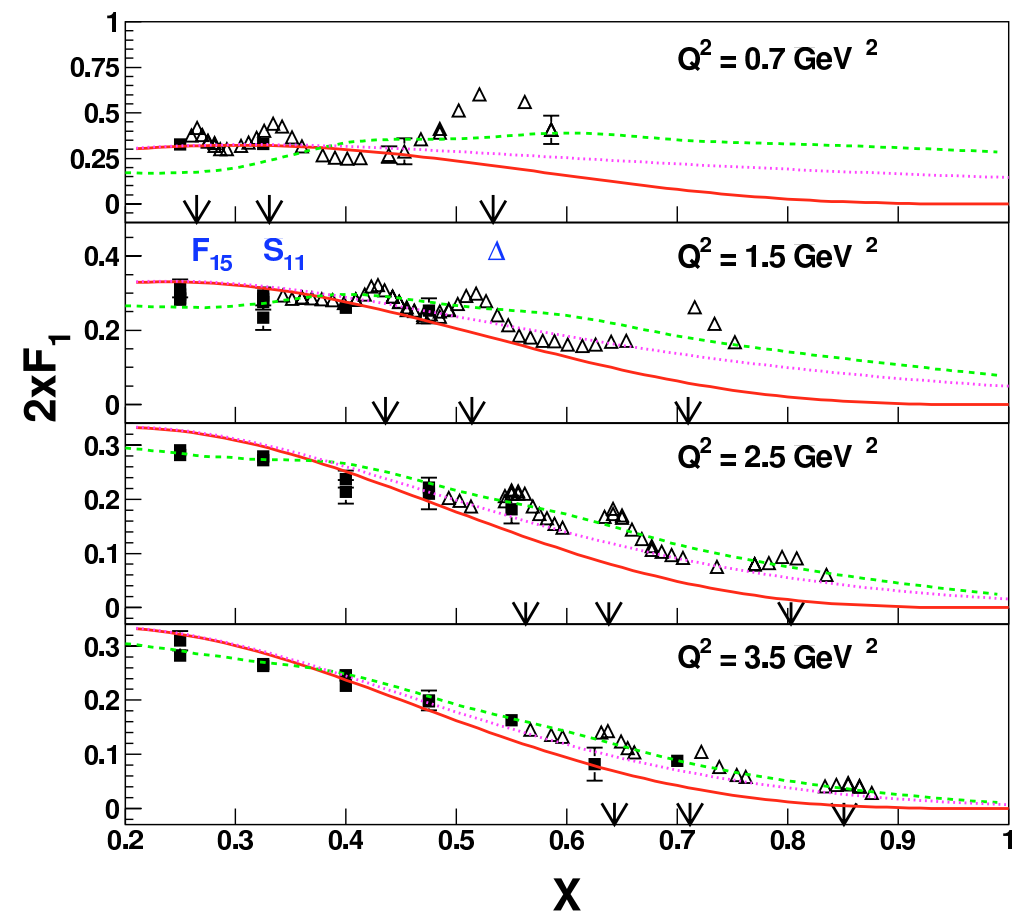


JLab Hall C

➡ TMCs significant at large x^2/Q^2 , especially for F_L

■ numerically...

non-zero at $x=1$!



JLab Hall C

➡ TMCs significant at large x^2/Q^2 , especially for F_L

Threshold problem

■ if $F(y) \sim (1 - y)^\beta$ at large y

then since $\xi_0 \equiv \xi(x = 1) < 1$

→ $F(\xi_0) > 0$

→ $F_i^{\text{TMC}}(x = 1, Q^2) > 0$

is this physical?

→ problem with GP formulation?

Possible solutions

■ Johnson/Tung - modified threshold factor

Nachtmann moment

$$\mu_2^n(Q^2) = \int_0^1 dx \frac{\xi^{n+1}}{x^3} \left(\frac{3 + 3(n+1)r + n(n+2)r^2}{(n+2)(n+3)} \right) F_2(x, Q^2)$$

→ n fixed, $Q^2 \rightarrow \infty$

$$\mu_2^n(Q^2) \rightarrow (\ln Q^2 / \Lambda^2)^{-\lambda_n} A_n$$

$$A_n = \int_0^1 d\xi \xi^n F(\xi)$$

→ $n \rightarrow \infty$, Q^2 fixed

$$\mu_2^n(Q^2) \rightarrow \xi_0^n(Q^2) \tilde{\mu}_2^n(Q^2)$$

“regularized” amplitudes
(threshold-independent)

Possible solutions

■ Johnson/Tung - modified threshold factor

Nachtmann moment

$$\mu_2^n(Q^2) = \int_0^1 dx \frac{\xi^{n+1}}{x^3} \left(\frac{3 + 3(n+1)r + n(n+2)r^2}{(n+2)(n+3)} \right) F_2(x, Q^2)$$

ansatz $\mu_2^n(Q^2) = \xi_0^n(Q^2) (\ln Q^2 / \Lambda^2)^{-\lambda_n} A_n$

➡ consistent with asymptotic pQCD behavior

➡ not unique!

Possible solutions

■ Johnson/Tung - modified threshold factor

moreover, if identify A_n with $M_2^n = \int_0^1 dx x^{n-2} F_2(x)$

$$\mu_2^n(Q^2) = \xi_0^n(Q^2) M_2^n(Q^2)$$

$$\rightarrow M_2^n(Q^2) = \mu_2^n(Q^2) + \frac{nM^2}{Q^2} M_2^n + \dots$$

cf. exact expression

$$M_2^n(Q^2) = \mu_2^n(Q^2) + \frac{n(n-1)}{n+2} \frac{M^2}{Q^2} M_2^{n+2} + \dots$$

\rightarrow inconsistency at low Q^2 ?

Possible solutions

- Kulagin/Petti - expand expressions in $1/Q^2$

$$F_2^{\text{TMC}}(x, Q^2) = \left(1 - \frac{4x^2 M^2}{Q^2}\right) F_2^{\text{LT}}(x, Q^2) + \frac{x^3 M^2}{Q^2} \left(6 \int_x^1 \frac{dz}{z^2} F_2^{\text{LT}}(z, Q^2) - \frac{\partial}{\partial x} F_2^{\text{LT}}(x, Q^2)\right)$$

Kulagin, Petti, NPA765 (2006) 126

➡ has correct threshold behavior

Alternative solution

■ work with ξ_0 dependent PDFs

→ n -th moment A_n of distribution function

$$A_n = \int_0^{\xi_{\max}} d\xi \xi^n F(\xi)$$

→ what is ξ_{\max} ?

- GP use $\xi_{\max} = 1$, $\xi_0 < \xi < 1$ unphysical
- strictly, should use $\xi_{\max} = \xi_0$

Alternative solution

■ what is effect on phenomenology?

→ try several “toy distributions”

standard TMC (“sTMC”)

$$q(\xi) = \mathcal{N} \, \xi^{-1/2} (1 - \xi)^3, \quad \xi_{\max} = 1$$

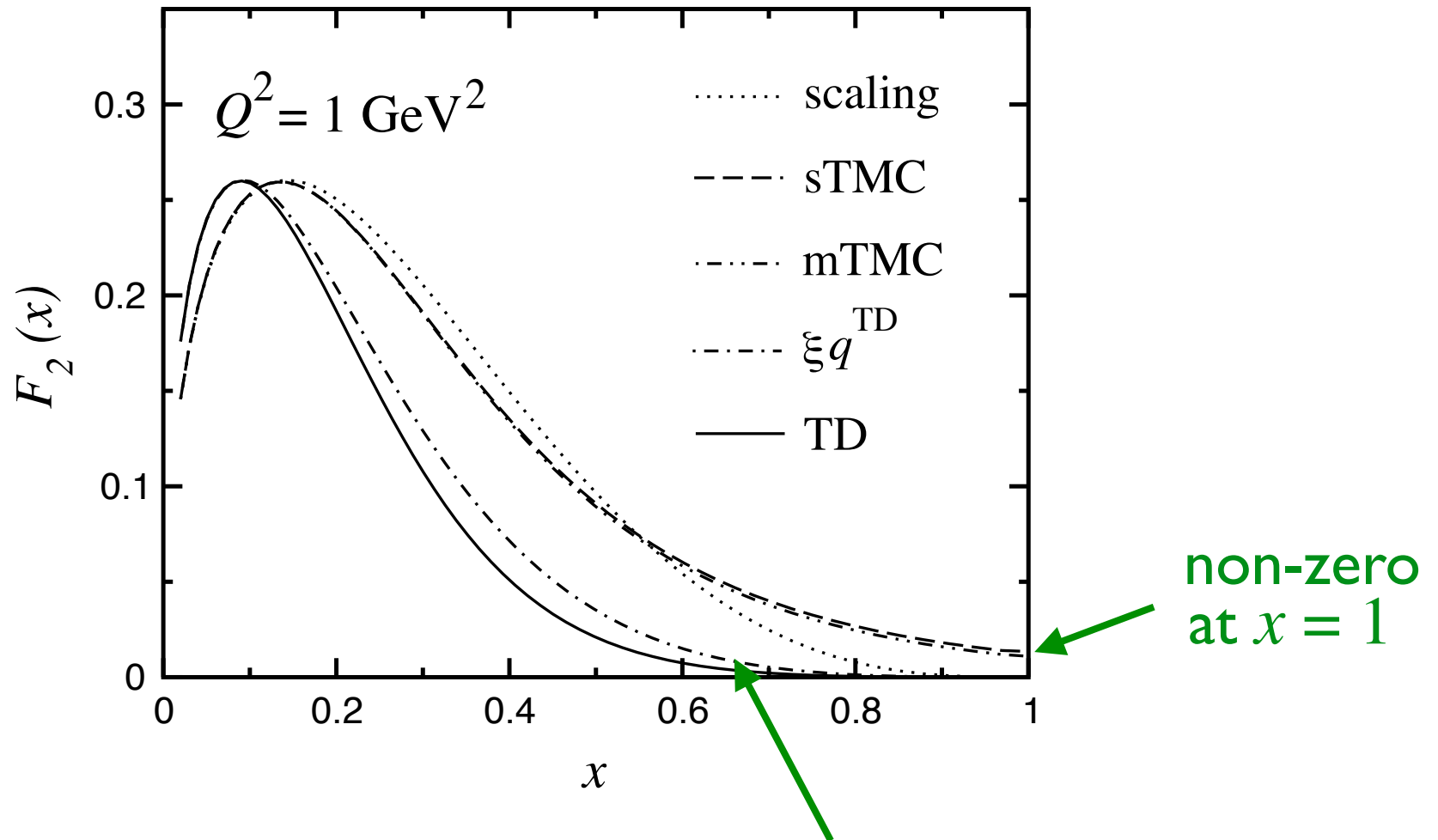
modified TMC (“mTMC”)

$$q(\xi) = \mathcal{N} \, \xi^{-1/2} (1 - \xi)^3 \Theta(\xi - \xi_0), \quad \xi_{\max} = \xi_0$$

threshold dependent (“TD”)

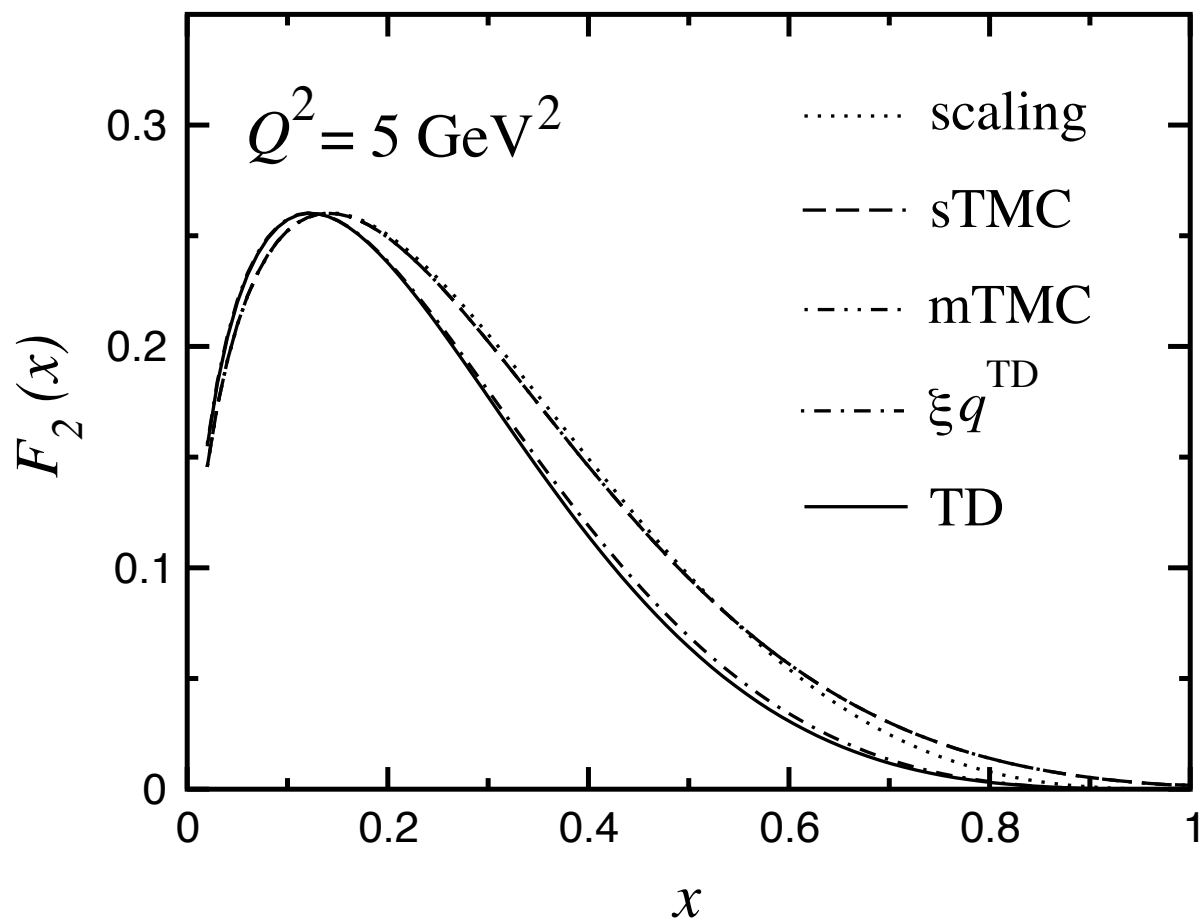
$$q^{\text{TD}}(\xi) = \mathcal{N} \, \xi^{-1/2} (\xi_0 - \xi)^3, \quad \xi_{\max} = \xi_0$$

TMCs in F_2



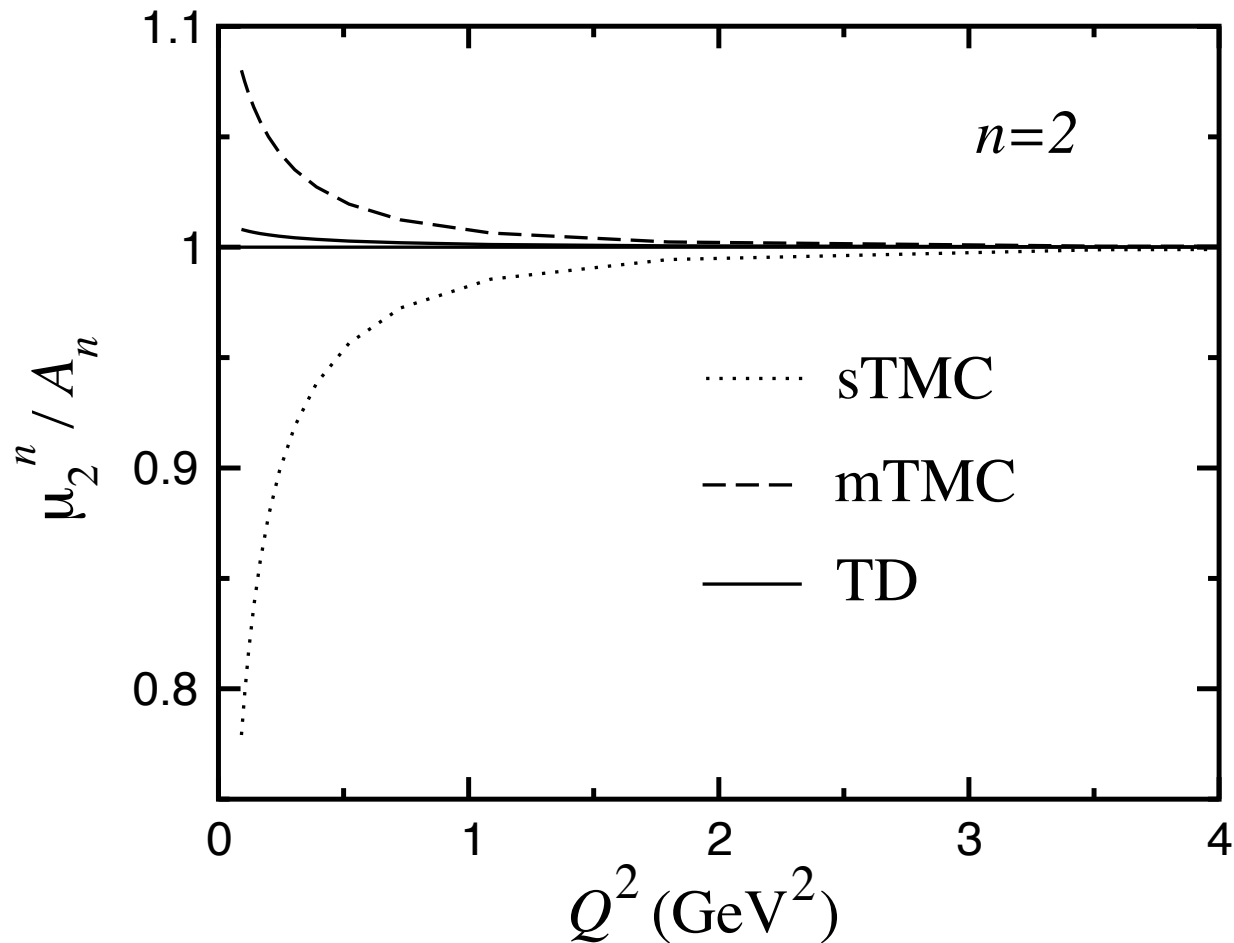
→ correct threshold behavior for “TD” correction

TMCs in F_2



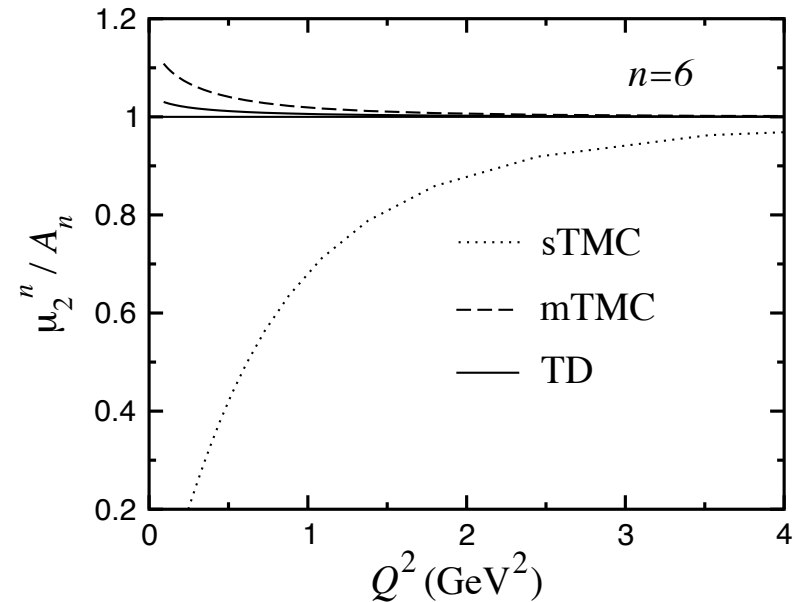
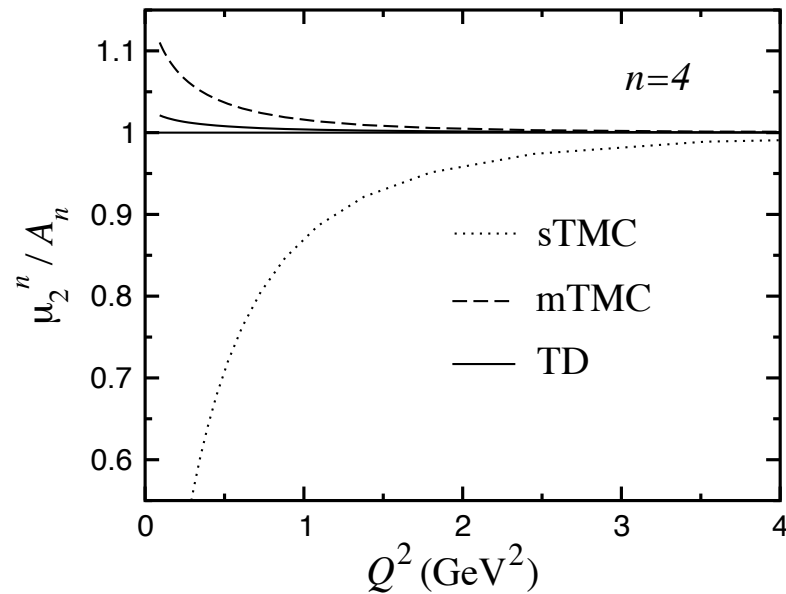
→ effect small at higher Q^2

Nachtmann F_2 moments



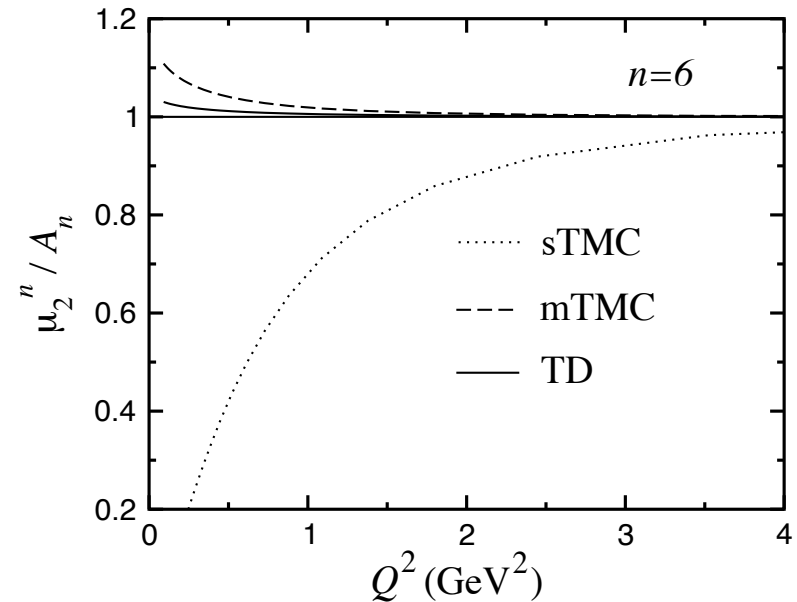
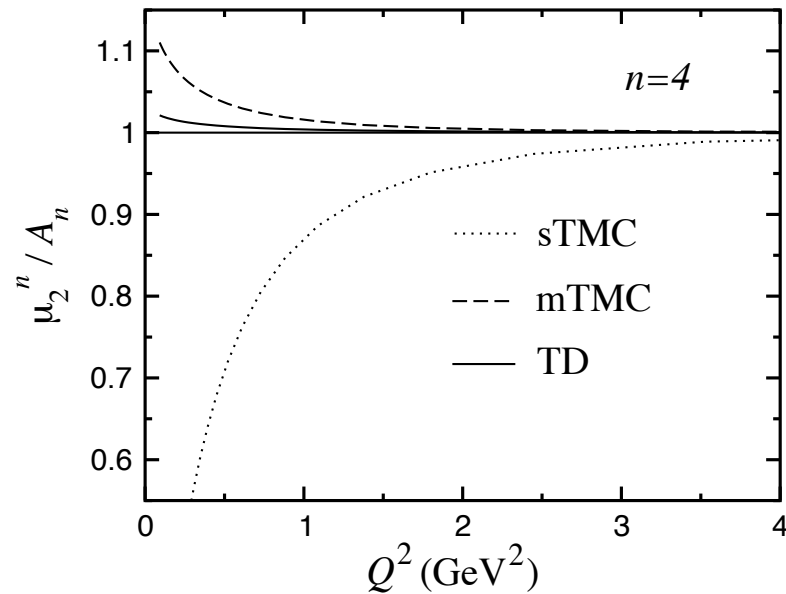
→ moment of structure function agrees with moment of PDF to 1% down to very low Q^2

Nachtmann F_2 moments



→ higher moments show much weaker Q^2 dependence than sTMC & mTMC prescriptions

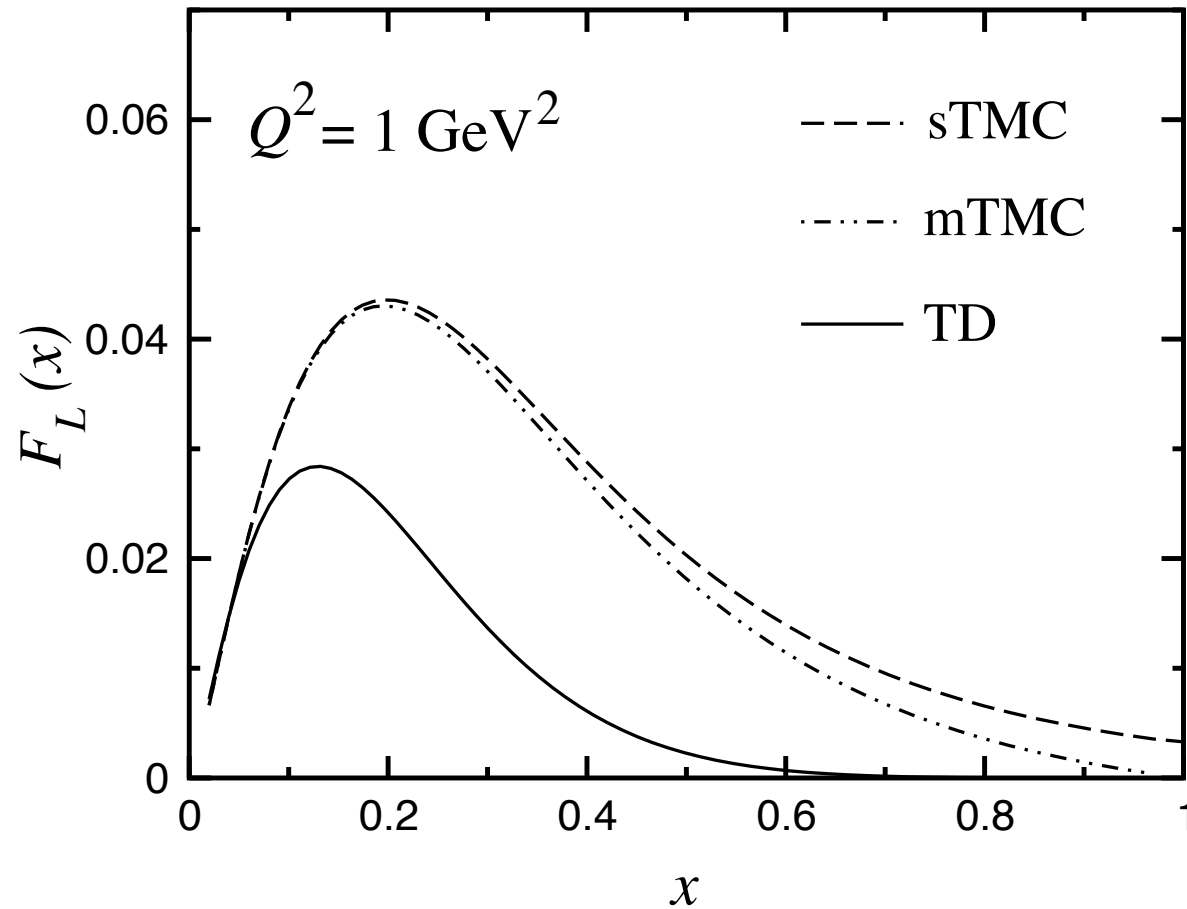
Nachtmann F_2 moments



→
$$\frac{\mu_2^n(\text{finite } Q^2)}{A_n(\text{finite } Q^2)} = \frac{\mu_2^n(Q^2 \rightarrow \infty)}{A_n(Q^2 \rightarrow \infty)}$$

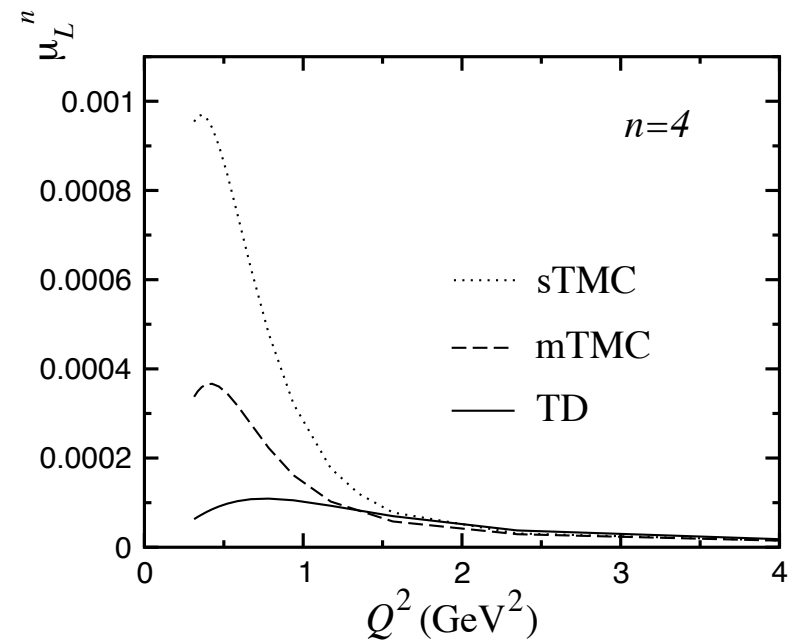
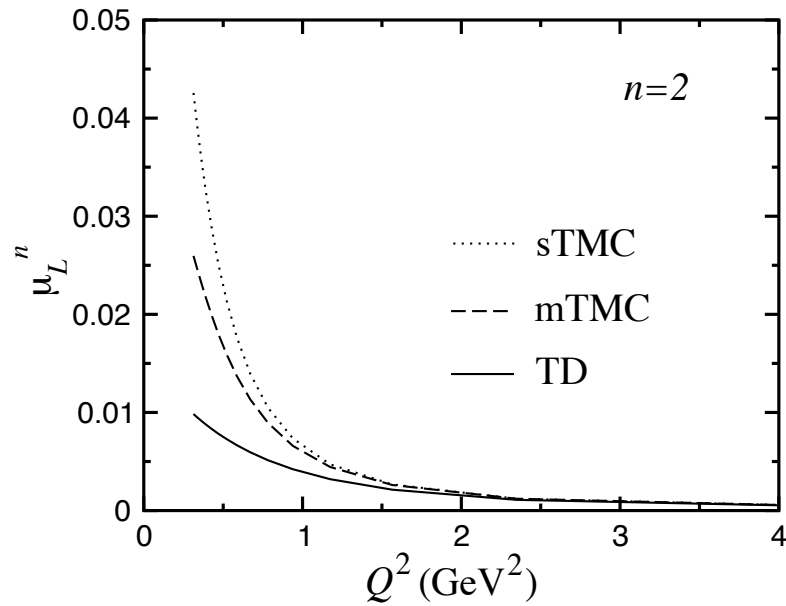
→ extract PDFs from structure function data at lower Q^2

TMCs in F_L



- correct threshold behavior for “TD” correction
- reduced TMC effect *cf.* sTMC and mTMC

Nachtmann F_L moments



→ weaker Q^2 dependence for TD prescription

Summary

- Target mass corrections important at low Q^2
- Formulations not unique
 - best known Georgi/Politzer prescription suffers from (unphysical) threshold problem at $x \sim 1$
- New formulation avoids threshold problem
 - much weaker Q^2 dependence of moments
 - introduces ξ and ξ_0 dependent PDFs
- Straightforward extension to neutrino structure functions